DOE Screening Time Trial

Pinewood Grand Prix

Lastufka Labs

Investigators:

Keywords: Pinewood, Grand Prix, DOE, Time trials, Speed factors

Abstract

16 March 2002 Primary: <u>Michael Lastufka</u> Assistant: <u>Arin Lastufka</u>

Ten pinewood Grand Prix car speed factors were screened using a two-level Design of Experiment (DOE) method. An eleventh, unused factor, was left in the experiment to help determine if there were significant interactions between factors. Selection criteria for the factors included potential for affecting speed and ease of modification to change between extreme configurations. A specially designed car allowed modification of each factor without disturbing the other factors.

All factors, with the exception of Wheel Base (B), were found significant with 95% confidence or better. Wheel Base exhibited borderline significance. Two factors, Coefficient of Friction (n) and Wheel Lifting (Lw) were shown to significantly reduce race time variation. Aerodynamics (A), n and Lw lead the list of greatest partial contributions to race time change. Wheel Mass (m), Wheel/Body Clearance (Cw), Nose Length (N), horizontal (CMf) and vertical (CMh) Center of Mass, Wheel Spinning before each race (St) and Wheel Base (B) were the other factors in this time trial. Analysis is complicated by the fact that wheel weighting and over-all stability[1] was dependent to some extent on horizontal center of mass. Thus, the investigators believe the order of the factors' partial effects are only approximate.

A few factors were selected based on the analysis to perform a three-level modeling Design of Experiment time trial. The mathematical model achieved will be compared to the investigator's detailed, closed-form model, documented in *Tracking Down Solutions* at http://www.worldforchrist.org/races/cars/why/summry.htm, and used to determine the relative strength of the effect of each factor more precisely.

Background

Pinewood car racing is beset by avid fans making many claims about race factors that are not supported by any science at all. At science fairs and on the Internet one can often find time trials and other results. Unfortunately, most neglect to design controls to isolate their trial factors or fail to document their projects well enough for independent verification - the hallmark of real science.

Advancement toward better experimentation includes at least two steps. First, the car(s) must be designed so that changes to speed factors affect other speed factors as little as possible. Kit axles generally cannot be removed and replaced without affecting wheel alignment, spacing and axle friction. Generally, weight cannot be moved or frontal cross-section changed without affecting each other. Some factors cannot be physically separated and must be dealt with in special ways.

Second, the car(s) must be configured so that *noise* (resulting in race time variation) is minimized to increase the possibility of significant results. Only statistically significant results obtained using proper controls have any scientific value.

However, if a body of well-performed experiments existed, it might not be as useful as one might expect. To obtain enough separation of factors and noise reduction for significant results, the specially designed car(s) and rigorous procedures may be unfit for direct application in competition. Differing race rules, tracks, kit materials, access to precision equipment and time constraints limit construction and race day choices for most participants. The car used in these time trials provides a simple example. It is too wide to pass a check-in inspection because of wheel collars that make changing wheels easy and "safe".

Nonetheless, rigorous time trials are useful to determine principles on which to design fast, legal cars. They also lend sanity checks to verify theoretical mathematical models that expose subtle interactions that cannot be economically measured on a non-existent budget.

The Design of Experiment (DOE) method promises to be a windfall in limiting the number of trials and runs needed to obtain rigorous results. For example, in this screening trial, ten factors are analyzed using only 12 different configurations and a total of 48 runs. It took an entire day to complete, but that's much less time then the 9 days previously anticipated to glean similar results.

Our experiments are being redesigned with the help of Randy Lisano, a certified Six Sigma Blackbelt, who is donating his expertise. This experiment is the first step of the two mentioned above. This *screening DOE* is designed to find significant factors that affect race time and race time variation. From those a set of five factors will be selected and the car(s) will be configured for reduced race time variation. A follow-up time trial, a *modeling DOE*, may produce useable equations from which optimal setting values for the trial factors can be found. Once prediction equations are derived for time and variation, runs will be conducted to determine if the equations' accuracy can be confirmed.

To find the ten speed factors for this Screening DOE time trial, we examined factors using the closed-form pinewood race model developed by the principal investigator. A few not included in the model were added to the list based on race experience. All these were sorted by expected reduction of race time considering reasonable *extreme values* for each factor. We assigned a *weight* to each factor on the sorted list indicating the difficulty of changing a trial car reliably to the extremes required for the factor. Another weight included a *popularity* rating based on data collected through the Lastufka Labs surveys. Combining these weights lead to another sort order. The top ten were selected. However, the factors on the final list of ten are not the only factors worthy of examination.

The table below presents the ten factors along with their extreme values and method of changing the factor value from the minimum to the maximum extreme.

Factors

Symbol	Name	Units	Minimum	Maximum	Method of Change
n	Friction Coefficient	scalar	0.02	0.2 [2]	Cleaned and lubed with silicone spray (low) vs. Dr. Pepper (high)
Lw	Wheel Lift	inches	0	0.1	Alignment jig vs. melted glue, lifted
m	Wheel Mass	ozs2/in	0.0001992	0.0008468 [3]	Offset weight vs. replace two wheels with inserted weights
Cw	Wheel Clearance	inches	0.00781	0.0625	Loosen wheel collars and tighten using wheel spacer
CMf	Center of Mass forward	inches	2.5	4.25	Main weight and offset weights positioned with car's peg system and masking tape
В	Wheel Base	inches	3.1875	5.3125	Positioned using car's wheel truck / peg system
N	Nose Length	inches	0.59375	1.5625	Positioned with car's front wheel truck / peg system
А	Frontal Cross- section	inches squared	2.60313	7.62656	Offset weight vs. hollow bulk frame
CMh	Center of Mass height	inches	1.2	2.375	Offset weight vs. main weight and offset weights placed on scaffold with masking tape
St	Wheel Spin time	seconds	2	10	Wheels spun with finger X times (once per second) before car placed on track
K	Unused Factor	scalar	-1	1	No procedure

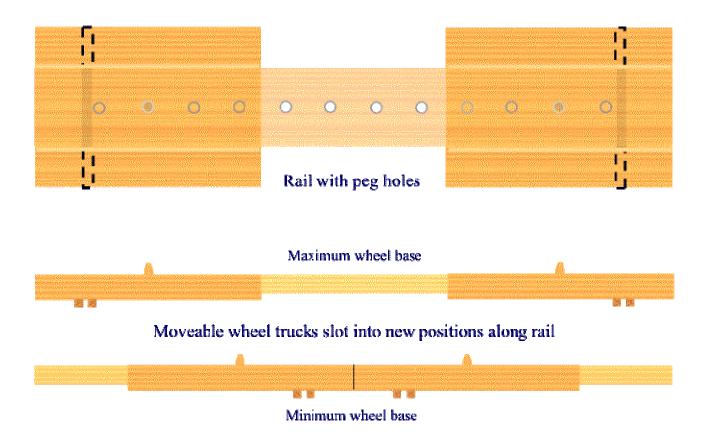
After determining the set of factor configurations, some of them turned out to be impossible with the original set of minimum and maximum values. For example a short-nose, short-wheel base configuration with rear weighting wheelied, dragging its rear end on the median. By correcting the factor extremes for these cases we arrived at the final values that appear in the Factor table above.

Unused factor (K):

The investigators believe the ten chosen speed factors to be the most important ones that can be tested economically. Leaving the unused factor in the experimental model, helps determine if there are significant interactions between pairs of factors (two-way interactions).

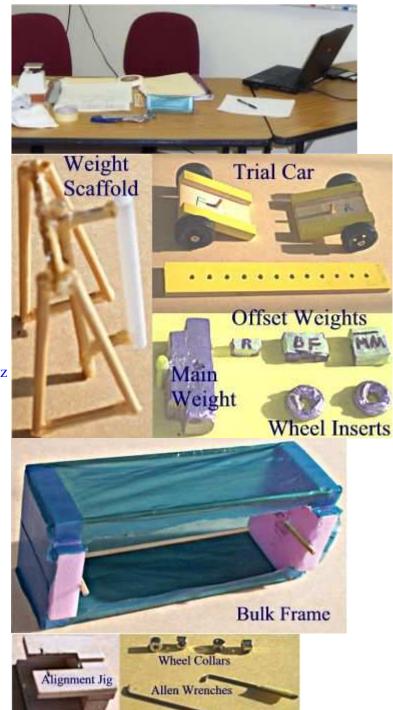
Setup

The list and pictures below show the main equipment and supplies needed to perform the screening DOE time trials. The track, time trial car and the accessories needed to achieve the various extreme factor configurations were built by the investigators. Most everything else was borrowed, including the laptop computer and digital camera - neither of which is required to duplicate this experiment.



The time trial car was built as pictured above. The illustration of the "minimum wheel base" is a possible extreme. It is not the one used in this time trial. It leads to impossible configurations with the other factors. The peg holes are spaced 1/2 inch center to center starting at 0.75 inches from each end.

Materials Heavy duty silicone spray Dr. Pepper (tm) Masking tape Lintless wipes Equipment Time trial track Fast Track digital timer Time trial car Main weight Alignment jig Wheel weight inserts 0.2 oz (2) Wheel spacers 1/128 in, 1/16 in Weight scaffold **Bulk Frame** Bulk frame weight offset 0.3 oz Wheel insert weight offset 0.4 oz Scaffold weight offset 0.1 oz Friction test ramp Tables for prep and alignment



Hot glue gun

Procedure

The DOE Screening Time Trials are based on a single five ounce car, changing its speed factor configurations and running it enough times to insure statistically valid results.

One practical strength of the DOE methodology allows mixed factor configurations. This greatly reduces the number of trials needed. Randy Lisano generated a Taguchi L-12 DOE matrix, which resulted in the set of 12 configurations shown in the Configuration table below for this screening DOE time trial. The change method indicated above in the Factor table was used to reconfigure the time trial car before each set of four runs.

Each factor is actually changed 6 times in the 12 configurations giving a total of 24 runs from which to determine specific effects of each factor when minimized and then when maximized (for a grand total of 48 runs).

Configurations

Each configuration is pictured at the left. The lower left end of the car faces forward. One can see some of the internal structure inside the semi-transparent bulk aerodynamic frame. The words in each cell correspond to a minimum (blue) extreme value for the column factor or a maximum (green).

Number	n	Lw	m	Cw	CMf	B	Ν	Α	CMh	St	K
<u>_</u> 1	Low	Down	Normal	Tight	Front	Short	Short	Small	Low	Few	Absent
<u>_</u>	Low	Down	Normal	Tight	Front	Long	Long	Large	High	Lots	Present
<u></u>	Low	Down	+0.2 oz	Loose	Rear	Short	Short	Small	High	Lots	Present
<u>_4</u>	Low	Lifted	Normal	Loose	Rear	Short	Long	Large	Low	Few	Present
5	Low	Lifted	+0.2 oz	Tight	Rear	Long	Short	Large	Low	Lots	Absent
6 <u>6</u>	Low	Lifted	+0.2 oz	Loose	Front	Long	Long	Small	High	Few	Absent

<u></u>	High	Down	+0.2 oz	Loose	Front	Short	Long	Large	Low	Lots	Absent
<u></u> 8	High	Down	+0.2 oz	Tight	Rear	Long	Long	Small	Low	Few	Present
9	High	Down	Normal	Loose	Rear	Long	Short	Large	High	Few	Absent
	High	Lifted	+0.2 oz	Tight	Front	Short	Short	Large	High	Few	Present
* 11	High	Lifted	Normal	Loose	Front	Long	Short	Small	Low	Lots	Present
<u>12</u>	High	Lifted	Normal	Tight	Rear	Short	Long	Small	High	Lots	Absent

Note the first six configurations and the last six required different lubricants. The car was lubed once with silicone spray before the first six, and it was lubed with Dr. Pepper before the last six. After the car was lubed, it was run several times until the time variation stabilized. The Dr. Pepper required many more runs as at one point it locked up the wheels before settling down. Most configurations ran smoothly without any obvious difficulties. A couple of the short wheel base configurations were rather "unstable" generating a couple spurious race times. These times, we threw out.

Most of the time between groups of runs was spent configuring the cars. This was not difficult. However, those configurations using the wheel weight inserts had to have the primary weightbearing pair of wheels removed from the car; the front wheels when front-weighted, the rear wheels otherwise. This operation took the most time of all of the procedures. A special Allen wrench fits in a tapping screw that holds a wheel collar on each axle. The collar acts the same as an axle head keeping the wheel on the axle. Using the collar allows the wheel to be removed with minimal disturbance to axle alignment and friction characteristics.

After each configuration change, we checked car alignment. Impacts with the track median and ballistic stoppers at the finish potentially alter alignment adversely. Placing it on a table top, we lifted one end of the table slightly to make the car roll slowly forward a few feet. Drifting from a straight line indicated that correction was needed. Corrections via melting hot glue around the axles and applying the alignment jig and a little finger pressure to set the jig were made a few times throughout the day. The jig was not needed with the lifted wheels as such fine alignment was not deemed critical.

Though over-all weight was not a factor, it was maintained at the same measure by offset weights when accessories were removed from the car. Maintaining the horizontal center of mass with changes in weight placement and offset weights was trickier. It was kept to within a quarter of an inch of the minimum and maximum CMf factor extreme values. This may have contributed to the detection of significant two-way interactions in the "unused" factor K.

Data collected on March-16-2002

The data below was collected by reading the times off the large display of the Microwizard, http://www.microwizard.com/, Fast Track timer[4] and entered into a computer file.

Configuration	Run 1	Run 2	Run 3	Run 4	Ave.	St.Dev.
01	2.864	2.868	2.882	2.872	2.872	0.00772
02	3.019	3.013	3.014	3.027	3.018	0.00640
03	2.913	2.920	2.924	2.911	2.917	0.00606
04	2.943	2.948	2.940	2.949	2.945	0.00424
05	2.946	2.950	2.946	2.953	2.949	0.00340
06	2.928	2.919	2.922	2.916	2.921	0.00512
07	3.330	3.308	3.481	3.440	3.390	0.08387
08	3.062	3.037	3.054	2.986	3.035	0.03413
09	3.166	3.159	3.062	3.180	3.142	0.05388
10	3.068	3.054	3.052	3.072	3.062	0.00998
11	2.921	2.935	2.918	2.898	2.918	0.01525
12	2.933	2.902	2.894	2.898	2.907	0.01780

Analysis

From the data a linear statistical model was constructed for both the average time (Y-hat model) and race time variation (S-hat model). The actual times and variations were subtracted from the predictions of these models to determine how far off they were. These differences are called "residuals". If the models are good, the actual measurements and noise are highly correlated with the model predictions. They were.

Y-Hat Model:

Multiple regression analysis was used to determine the statistical significance of each factor's

effect on the output (time). From the P(2 Tail)'s, which is the probability of a factor NOT being significant, all factors, with the exception of Wheel Base (B) were found to be statistically significant. The confidence level in these results is 95%. Wheel Base was a borderline case, being marginally significant.

Even the unused factor K was shown to be statistically significant. So, what is in the *unused* factor K? With the Taguchi L-12 design matrix used in this experiment, significant two-way interactions between factors would result in an unused factor being statistically significant. For this design matrix, all effects from two-way interactions are confounded with (hidden among) the main effects of each factor, but this confounding is equally spread across all factors. Since K was not a used factor, there was no main

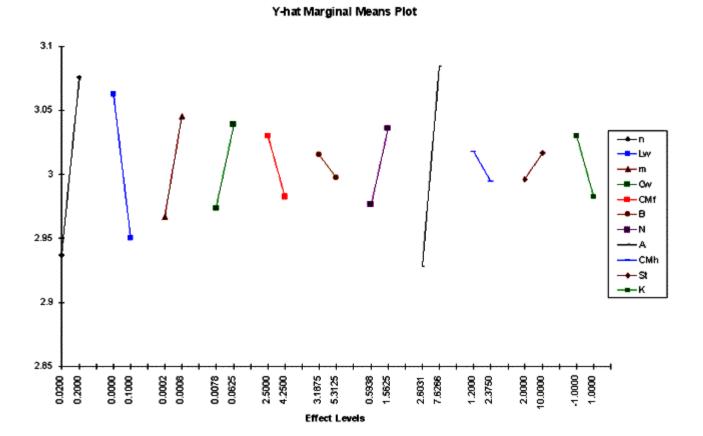
	Y-hat Model										
Factor	Name	Coeff	P(2 Tail)	Tol	Active						
Const		3.00619	0.0000								
A	n	0.06923	0.0000	1	Х						
В	Lw	-0.05598	0.0000	1	Х						
С	m	0.03931	0.0000	1	Х						
D	Cw	0.03260	0.0000	1	Х						
E	CMf	-0.02385	0.0000	1	Х						
F	В	-0.00906	0.0542	1	Х						
G	Ν	0.02977	0.0000	1	Х						
Н	А	0.07798	0.0000	1	Х						
I	CMh	-0.01177	0.0139	1	Х						
J	St	0.01023	0.0309	1	Х						
K	К	-0.02377	0.0000	1	Х						
Rsq	0.9621										
Adj Rsq	0.9505										
Std Error	0.0316										
F	83.1206										
Sig F	0.0000										
Source	SS	df	MS								
Regression	0.9	11	0.1								
Error	0.0	36	0.0								
Total	0.9	47									

effect to confound with, so only significant two-way interactions would result in K being significant. The wheel weight insert dependency on CMf might be the cause, but since all two-way interactions are confounded equally in K, there is no way to determine if there are any other significant two-way interactions in these experimental results. Other two-way interactions include a synergism of nose length, wheel base and weight placement that is known to affect overall car stability. A combination like wheel spacing and track roughness may produce other synergies.

Additional "sanity checks" were made by examining the residuals and using ANOVA analysis. From the R² and Adjusted R² results, being over 95% and having values that were very close to each other, the fit of the experimental data using linear regression was very good. From ANOVA analysis, we get the *standard error* of the data, the *Fischer ratio* (F), and another measure of significance. The standard error measures how accurately the mean (average) race times were determined by the experiment. It was very small, so the means are very accurate. The Fisher Ratio indicates if there are any factors in our model that are significant, but it won't pinpoint which ones. When F is greater than 6, there is a statistically significant factor. Ours was 83, confirming that something was indeed significant. The third measure is the probability that none of the factors were significant (Sig F). This one was very near zero, confirming again that there was at least one significant factor in the model.

Since this was a Screening DOE, no prediction equation can be derived. However, from the magnitudes of the coefficients, the top factors can be determined for use in a follow on Modeling DOE. If all goes well with the Modeling DOE, a useable prediction equation can be derived that can be used to confirm the accuracy of the resulting model.

Four plots were created, *Y-Hat* a plot of effect on average race time, *Y-Hat Pareto* an ordered view of half Y-Hat values, *S-Hat* the effect on race time variation and the ordered *S-Hat Pareto* view, to graphically compare the effects of the experimental factors.



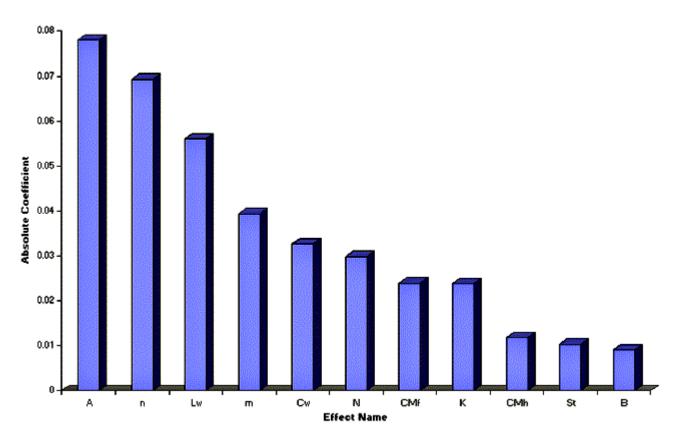
Y-Hat Marginal Means Plot:

The Y-hat Marginal Means Plot graphically shows the effect of extreme changes in the factor (each factor has a min and max measurement on the x-axis) on race time (y-axis). Let's look at the friction coefficient "n". The minimum value on the x-axis is 0.02, which has a race time of 2.937 seconds. The maximum value, 0.2, (next point to the right on the x-axis) has a race time of 3.075 seconds. A line is drawn between them. The lower friction coefficient value has a lower race time, just as you would expect. Note that the distance the line covers in the y direction is large compared to most of the other lines. This factor was found to have a large effect on race time.

Lw for lifting a wheel can be read similarly, but it shows the opposite effect. Its minimum value, zero, is achieved when the wheel touches the track. Its maximum value, 0.1 inch, is above the track. Presumably, a lower value like 0.05 inches could allow the wheel to momentarily touch the track and slow the car. Note, that lifting the wheel (the maximum value

for Lw) had a lower race time than when it was on the track (zero) and that the spread on the yaxis is nearly as large as the one for friction coefficient.

For any line representing a factor, the lower end is above the factor value that improved the car's race time.



Y-Hat Pareto of Coefficients:

The Y-Hat Pareto of Coefficients chart plots half the y-axis spread from the Y-Hat Marginal Means Plot in order of largest spread to smallest. In this chart, it is easy to see which factors had the greatest effect on race time - but you can't tell whether the effect slowed the car down or sped it up. Look at the "A" and "m" bar. It indicates that the bulk frame and wheel weight inserts had measurable effects on the race time. From the Y-Hat Marginal Means Plot, one can see they slowed the car down, increasing race time. Don't add bulk (frontal area) to a car or weight to wheels on a 30-foot down sloping track!

Y-hat Pareto of Coeffs

Symbol	Α	n	Lw	m	Cw	N	CMf	К	CMh	St	В
Name	Frontal Cross- section	Friction Coefficient	Wheel Lift	Wheel Mass	Wheel Clearance	Nose Length	Center of Mass forward	Unused Factor	Center of Mass height	Wheel Spin time	Wheel Base
Best Setting	Small	Low	Lifted	Normal	Tight	Short	Rear	Present	High	Few	Long
Advantage (sec)	0.078	0.069	0.056	0.039	0.033	0.030	0.024	0.024	0.012	0.010	0.009

Y-hat Advantage mean = 3.0061875 seconds

In the table above, all the factors are listed with their average race time advantage and best extreme setting. Other settings may produce optimal race time reduction but that's what we hope to find out in the follow-up time trial! Also, note how the unused factor put in a fairly good showing. Since two-way interactions are confounded among the main factor effects, the follow-on modeling DOE will be needed to determine the magnitude of each factor's effect, and the effects of any interactions, on the output (time).

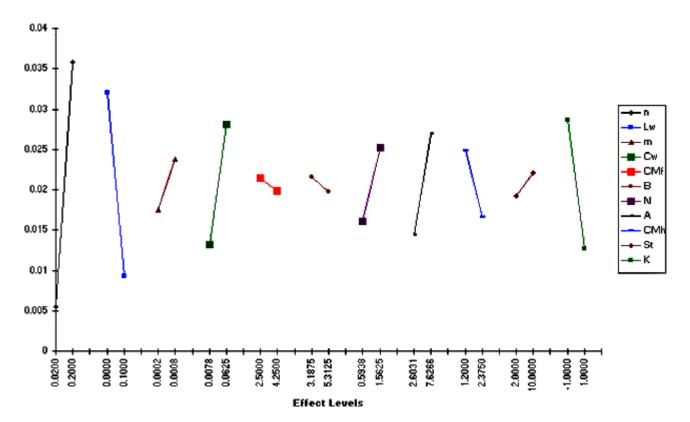
S-Hat Model:

Multiple regression analysis was also used to determine the statistical significance of each factor's effect on time variation. For a two-level design, such as used in this experiment, a rule of thumb for determining statistical significance can be applied. This rule of thumb states that the absolute value of a factor's coefficient must be greater that half the constant in order to be considered significant. The only two factors that met this rule of thumb were Friction Coefficient (n) and Wheel Lift (Lw), with a confidence level of 95%.

Again, since this was a Screening DOE, no prediction equation can be derived and no further analysis can be made of the strength of the S-hat model.

		S-hat Model			
Factor	Name	Coeff	P(2 Tail)	Tal	Activo
	Name			101	Active
Const		0.02066	Not Avail		X
A	<u>n</u>	0.01516	Not Avail	1	Х
B	Lw	<mark>-0.01135</mark>	Not Avail	1	Х
C	m	0.00311	Not Avail	1	Х
D	Cw	0.00742	Not Avail	1	Х
E	CMf	-0.000736878	Not Avail	1	Х
F	В	-0.000957976	Not Avail	1	Х
G	Ν	0.00461	Not Avail	1	Х
Н	А	0.00631	Not Avail	1	Х
I	CMh	-0.00412	Not Avail	1	Х
J	St	0.00148	Not Avail	1	Х
K	K	-0.00798	Not Avail	1	Х
Rsq	1.0000				
Adj Rsq	Not Avail				
Std Error	Not Avail				
F	Not Avail				
Sig F	Not Avail				
3 -					
Source	SS	df	MS	1	
Regression	0.0	11	0.0	1	
Error	0.0	0	Not Avail		
Total	0.0	11			

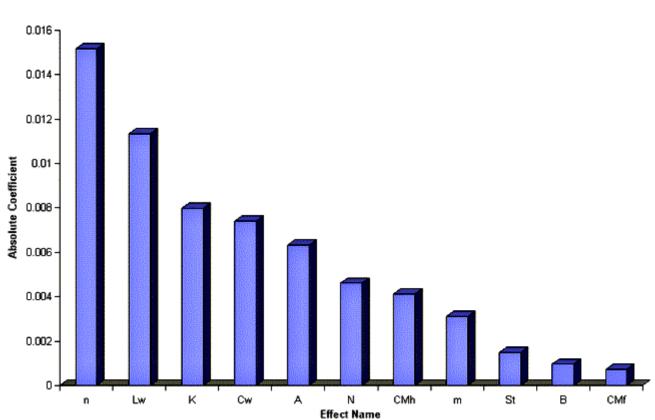
S-Hat Marginal Means Plot:



S-hat Marginal Means Plot

Y-axis values on the S-Hat Marginal Means Plot center around 0.02066 which is more-or-less an average deviation for all race time measurements. The deviation in the measurements resulting from each factor being minimized and then maximized is plotted as a line. The lower S-hat values indicate less variation in race time. These values give clues as to which factors help "stabilize" a pinewood car. Significant factors should have a spread greater than the S-hat center value (0.02066). Champion cars must increase stability and top speed. Low friction and lifting a wheel helped to make the trial car more stable. "K", the unused factor, is small enough to conclude that there were no significant two-way interactions between factors that affected the car's stability. However, a combination like wheel spacing and track roughness may yet prove to have a significant effect.

S-Hat Pareto of Coefficients:



S-hat Pareto of Coeffs

The S-Hat Pareto of Coefficients chart plots half the y-axis spread from the S-Hat Marginal Means Plot in order of largest spread to smallest. It is easy to see which factors had the greatest effect on race time variation – but, from this graph, you can't tell whether the effect was stabilizing or destabilizing.

Additional "sanity checks" were made using ANOVA (analysis of variance) analysis. From ANOVA analysis, we get the *standard error* of the data, the *Fischer ratio* (F), and another measure of significance. The standard error measures how accurately the mean (average) race times were determined by the experiment. It was very small, so the means are very accurate. The Fisher Ratio indicates if there are any factors in our model that are significant. It won't pinpoint which ones. When F is greater than 6, there is likely a significant factor. Ours was 83, confirming that something was indeed significant. The third measure is the probability that none of the factors were significant (Sig F). This one was very near zero, confirming again that likely there were no insignificant factors.

Results

Within reason, we can be 95% confident that all factors, with the exception of Wheel Base, significantly affected the race time of the car. Additionally, we can be 95% confident that Friction Coefficient (n) and Wheel Lift (Lw) had a statistically significant effect on time variation. The DOE screening method also gave an indication of how much each factor changed the race time on the average (Y-Hat Advantage). However, these should not be taken as the final measure of how the factors stack up to each other. If this experiment were run again, some in the order might well change, but highs would likely still be high and the lows still low.

Low friction means that surface rubbing has less of an effect than it would otherwise; fewer sticky areas, bumps, ridges, plastic filaments, etc., to cause a wheel to shift position on its axle. When a wheel runs true, it is less likely to cause the car to wobble or drift from a straight path down the track. That's *stability*.

Put four wheels on the track and there are four independent sources of destabilizing, jostling forces. Lift one of them off the track and now there are only three wheels to destabilize the run. Stability is improved by lifting a wheel! This experiment measured that effect and showed it to be significant. Getting more consistent times is enough reason to lift a wheel, but the analysis also showed that this practice speeds up the car too!

Further Investigation

Many follow-up trials are possible to further explore the details of these results and to find new ones. But this experiment and all subsequent ones should be duplicated elsewhere. Our hope is that this report contains everything necessary to reproduce this screening study and that other investigators would take on the challenge of confirming or refuting our results.

The over-all results of this experiment agree well with the investigators' mathematical models. In the sequel, the DOE method will be applied to produce an empirical mathematical model of 5 factors. It should be possible to derive that empirical model as a special case of the more general theoretical model. If not, it may yield new insights into the interactions of the factors and possibly show that other factors (known or unknown) mediate the dynamics.

As of this report, the factors will be:

- 1. Position of weight (front to rear CMf)
- 2. Frontal Cross-section
- 3. Body-Bore Clearance
- 4. Nose Length
- 5. Wheel Base

For each of these five factors, three settings will be used including the extremes as in this screening study; the other would be the midpoint setting. All configurations will include low

friction, lifted wheel and low weight wheels (cut). These choices greatly simplify the time trial procedures and eliminate the need to remove wheels and replace them.

Notes

- [1] "Stability" is the way a vehicle travels along a path. Does it jostle away from the path or spin, bob up and down or roll from side-to-side as it moves? A car that "snakes" down the track, twisting from one side of the lane median to the other is not very stable. One that only drifts to one side is more stable. A car that runs straight with no wheel vibration or wobble or noticeable deflection from the lane median or track surface is very stable.
- [2] As of this report, there is at least one "loose-end". Nominal values were used for the coefficient of axle friction. The actual values were measured by the investigators but have not been calculated from the raw data. The two values should be available later this summer.
- [3] Later the wheel weight inserts were weighed more accurately and found to weigh 0.2 ounces each instead of the target 0.25 oz. Thus the mass of a weighted wheel is 0.0007172 ozs2/in.
- [4] Purchase options included a computer link, but we did not order it. The timer was purchased for Lastufka Labs by Dan Kolsar and Jeff Heath at a special price. Thanks Microwizard, it's a great timer!

Reference:

Schmidt, Stephen R. and Robert G. Launsby. Understanding Industrial Designed Experiments, 4th Edition, Air Academy Press, Colorado Springs, CO, 1994.

Schmidt, Stephen R. and Ken Case, PhD. Communicating Design of Experiments (DOE) to Non-statisticians, <u>Air Academy Associates</u>, <u>http://www.airacad.com/comdoe.htm</u>, LLC, (719) 531-0777. 17 May 2002.