DOE Modeling Time Trial

Pinewood Grand Prix

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Abstract

This time trial pitted five factors, wheel/body clearance (Cw), Frontal Cross-section (A), Nose Length (N), Wheelbase (B), and Center of Mass measured from the front (CMf), directly against one another and in paired combination using five settings each. Leveraging Design Of Experiment (DOE), simple mathematical models of these pinewood car performance factors emerged for two time trial cars in 30 configurations. In particular, the models detail how each factor or pair of factors contributes to speed and stability. Specially designed cars enabled factor isolation for adequate reduction of race time variation. Each factor passed a previous Screening DOE Time Trial[1].

Two final sets of runs with a new configuration checked the veracity of the models. They indicated along with statistical measures that the blue car model is a better predictor of actual times than the yellow model. Most of the blue car data fit noiseless theoretical time predictions slightly better than the actual average time model and both accurately predicted the 30th configuration race time.

The DOE method provided relatively simple models of both the noiseless theoretical races and the actual races. Comparison shows instabilities working through wheel play, and using the nose and wheelbase as levers. Air resistance proved a stronger factor than previously thought and wheel play insignificant. The factors of nose length, wheelbase and center of mass demonstrated interdependence and regions of instability. Some configurations should be avoided as unstable; notably ones with short wheelbases and front-weighted configurations with no short nose and long wheelbase.

Background

This experiment follows the DOE Screening Time Trial of March 16, 2002. Of the ten pinewood car speed factors found to be significant, five were selected for this modeling study. 29 configurations of three runs each gathered data on five settings for each factor; a five factor, three-level experiment design. Originally, this experiment ran on August 17, 2002. Unfortunately, the two data sets collected were too noisy to produce a valid model. Some of the noise originated from CMf settings too far rearward and forward. The trial cars became so unstable that configurations such as N=short, B=short and CMf=aft or rear often failed to cross the finish line. With adjusted CMf settings and a new set of wheels prepared using the Maximum Velocity *Pro Hub* wheel tool, the November 2, 2002 time trial produced two valid models.

In the DOE Screening Time Trial, it was noted that a few of the trial car configurations were not as stable as most. The unstable configurations arose from combinations of nose, base and center of mass settings. Thus, the three factors couple to some degree in unstable car configurations. This experiment showed the interdependence of such effects.

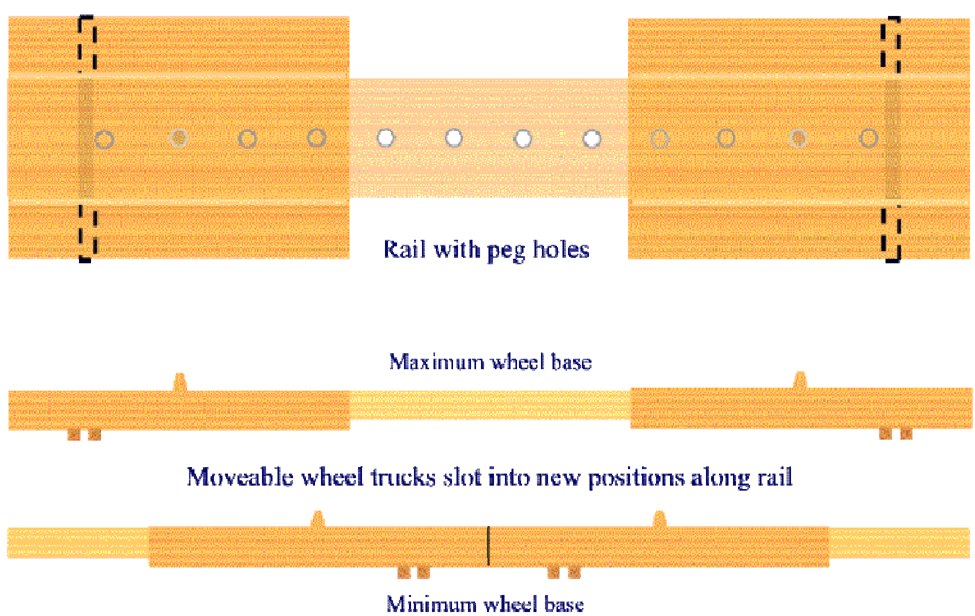
The table below presents the five factors along with their five settings and method of changing the factor settings.

Factors

Symbol	Name	Units	Minimum	Low	Normal	High	Maximum	Method of Change
Cw	Wheel Clearance	inches	Tightest 1/64	Tight 1/32	Normal 3/64	Loose 1/16	Loosest 5/64	Loosen wheel collar, insert wheel spacer and re-tighten
A	Frontal Cross-section	inches squared	Smallest 2.6	Small 3.85	Normal 5.1	Large 6.35	Largest 7.7	Plug baffle into rear of car rail
N	Nose Length	inches	Shortest 0.6	Short 1.1	Normal 1.6	Long 2.1	Longest 2.6	Positioned with car's front wheel truck / peg system
B	Wheelbase	inches	Shortest 3	Short 3 7/8	Normal 4 3/8	Long 4 7/8	Longest 5 3/8	Positioned using car's rear wheel truck / peg system
CMf	Center of Mass forward	inches	Front 1 7/8	Fore 2 5/16	Center 3 3/4	Aft 4 15/16 +/- 1/16	Rear 5 7/8	Main weight positioned with car's peg system and masking tape

Setup

The list and pictures below show the main equipment and supplies needed to perform the modeling DOE time trials. The track, time trial cars and the accessories needed to achieve the various factor configurations were built by the investigators. Most everything else was borrowed, including the digital camera - which is not required to duplicate this experiment.



The time trial cars were built as pictured above. The illustration of the "minimum wheelbase" is a possible extreme. It is not one used in this time trial. It leads to impossible configurations with the other factors. The peg holes are spaced 1/2 inch center to center starting at 0.75 inches from each end.

Materials

Heavy duty silicone spray

Graphite with molybdenum

Masking tape

Lintless wipes

Equipment

Time trial track

Fast Track digital timer

2 Time trial cars

4 Thin wheels (1/8 inch tread)

4 Very thin wheels (1/16 inch tread)

2 Main weights

Alignment jig for axles

Wheel spacers 1/64 in, 5/64 in

Tables for prep and alignment

Hot glue gun

Pro-Hub Tool http://www.maximum-velocity.com/pro-hub.htm

Procedure

The DOE Modeling Time Trials are based on two data sets measured independently for two five-ounce cars, changing speed factor configurations and running them enough times to insure statistically valid results.

One practical strength of the DOE methodology allows mixed factor configurations. This greatly reduces the number of trials needed. Randy Lisano generated this set of 29 configurations shown in the Configuration table below for this modeling DOE time trial. The change method indicated above in the Factor table was used to reconfigure the time trial cars before each set of three runs.

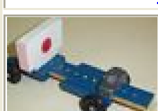
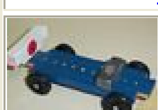
Each factor is actually changed several times in these configurations giving many runs from which to determine specific effects of each factor and their interactions.

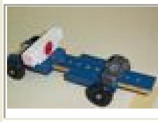
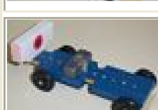
The blue car's thin wheels (half tread cut off) were lubricated the night before the trial with silicone spray and not lubed again. Every three runs, the yellow car's one-quarter width wheels were lubricated with graphite containing molybdenum. Preliminary trials indicate that the performance of graphite with molybdenum degrades immediately after application, improves after a few runs, then degrades again. Running three times after application insured less time variability though over-all speed decreases. This time trial is not able to compare the two lubricants accurately.

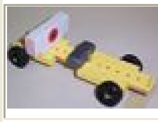
The truck of the lifted wheel of each car was placed in the front position or rear position; which ever was farther from the center of mass. This kept the cars level and as stable as possible. Shifting positions necessitated rotating the truck 180 degrees. It was noted that swapping trucks front to rear on the blue car had no observable effects, it remained more-or-less in the center of its lane. But the yellow car switched from lightly nudging the right side of the lane median to the left.

Configurations

Each configuration is pictured at the left. These pictures are from the August 17, 2002 trial. In this trial the yellow car had cut wheels thinner than the blue car's. The front, right end of each car faces forward. The background of each cell corresponds to a pink minimum setting, blue low setting, gray normal setting, green high setting or yellow maximum setting for the factor. In the interest of time, each of the three repetitions of a configuration was run one after the other. Configurations of each car were scheduled depending on the accessories required to race both cars together.

Number	Cw	A	N	B	CMf
 1	Tight	Small	Short	Short	Aft
 2	Tight	Small	Short	Long	Fore
 3	Tight	Small	Long	Short	Fore
 4	Tight	Small	Long	Long	Aft
 5	Tight	Large	Short	Short	Fore
 6	Tight	Large	Short	Long	Aft
 7	Tight	Large	Long	Short	Aft
 8	Tight	Large	Long	Long	Fore
 9	Loose	Small	Short	Short	Fore

 10	Loose	Small	Short	Long	Aft
 11	Loose	Small	Long	Short	Aft
 12	Loose	Small	Long	Long	Fore
 13	Loose	Large	Short	Short	Aft
 14	Loose	Large	Short	Long	Fore
 15	Loose	Large	Long	Short	Fore
 16	Loose	Large	Long	Long	Aft
 17	Normal	Normal	Normal	Normal	Center
18	Normal	Normal	Normal	Normal	Center
19	Normal	Normal	Normal	Normal	Center
 20	Tightest	Normal	Normal	Normal	Center
 21	Loosest	Normal	Normal	Normal	Center
 22	Normal	Smallest	Normal	Normal	Center
 23	Normal	Largest	Normal	Normal	Center
 24	Normal	Normal	Shortest	Normal	Center

 25	Normal	Normal	Longest	Normal	Center
 26	Normal	Normal	Normal	Shortest	Center
 27	Normal	Normal	Normal	Longest	Center
 28	Normal	Normal	Normal	Normal	Front
 29	Normal	Normal	Normal	Normal	Rear
 30	Tight	Smallest	Shortest	Longest	Rear

Note that configurations 17, 18 and 19 are identical. These represent *normal* settings. There are three so that one could be run at the beginning of the trials, one between the "high/low" setting configurations and the "max/min" setting configurations and one at the end. They help indicate any bias affecting the times over the course of the time trials.

As alluded to above, the configurations break out into two main groups. The "high/low" setting configurations (blue and green) build a profile of factor change. By integrating the profile with the "max/min" setting configurations (yellow and pink), a profile of factor vs. factor influence appears. These are expressed in the linear and non-linear terms of the model of mean times (y-hat model).

A model check configuration, different than the others became the 30th configuration. This configuration models a very fast pinewood car.

Data collected on November-2-2002

Data collected for the **BLUE car** by reading the times off the large display of the Microwizard, http://www.microwizard.com/, Fast Track timer[2] appears below.

Configuration	Run 1	Run 2	Run 3	Ave.	St.Dev.
01	2.939	2.936	3.029	2.968	0.0528
02	2.903	2.905	2.909	2.906	0.0031
03	2.940	2.951	2.929	2.940	0.0110
04	2.882	2.881	2.885	2.883	0.0021
05	3.003	2.981	2.988	2.991	0.0112
06	2.934	2.928	2.932	2.931	0.0031
07	2.933	2.937	2.933	2.934	0.0023
08	2.979	3.014	2.974	2.989	0.0218
09	2.916	2.949	2.939	2.935	0.0169
10	2.887	2.886	2.891	2.888	0.0026
11	2.880	2.880	2.875	2.878	0.0029
12	2.921	2.930	2.930	2.927	0.0052
13	3.047	3.093	3.025	3.055	0.0347
14	2.972	2.973	2.958	2.968	0.0084
15	3.013	2.999	3.012	3.008	0.0078
16	2.934	2.938	2.937	2.936	0.0021
17	2.911	2.906	2.912	2.910	0.0032
18	2.910	2.916	2.914	2.913	0.0031
19	2.906	2.910	2.904	2.907	0.0031
20	2.908	2.912	2.915	2.912	0.0035
21	2.904	2.905	2.904	2.904	0.0006
22	2.851	2.852	2.852	2.852	0.0006
23	2.981	2.980	2.982	2.981	0.0010
24	2.922	2.929	2.929	2.927	0.0040
25	2.928	2.932	2.930	2.930	0.0020
26	2.938	2.925	2.936	2.933	0.0070
27	2.908	2.904	2.901	2.904	0.0035
28	2.963	2.967	2.955	2.962	0.0061
29	2.915	2.904	2.911	2.910	0.0056

Data collected on November-2-2002

Data collected for the **YELLOW car** by reading the times off the large display of the Microwizard, Fast Track timer appears below. An error in data entry was corrected for configuration 26 run 2 from 2.757 seconds to 2.957.

Configuration	Run 1	Run 2	Run 3	Ave.	St.Dev.
01	3.124	3.209	3.284	3.206	0.0801
02	3.018	3.013	3.010	3.014	0.0040
03	3.013	3.168	3.058	3.080	0.0797
04	2.982	2.986	2.984	2.984	0.0020
05	3.078	3.080	3.070	3.076	0.0053
06	3.110	3.103	3.084	3.099	0.0135
07	3.064	3.061	3.076	3.067	0.0079
08	3.098	3.083	3.092	3.091	0.0075
09	2.993	2.992	2.987	2.991	0.0032
10	2.935	2.950	2.947	2.944	0.0079
11	2.946	2.944	2.931	2.940	0.0081
12	2.993	3.006	3.002	3.000	0.0067
13	3.227	3.092	3.241	3.187	0.0823
14	3.062	3.066	3.068	3.065	0.0031
15	3.070	3.063	3.052	3.062	0.0091
16	2.970	2.978	2.975	2.974	0.0040
17	3.034	3.031	2.998	3.021	0.0200
18	3.034	3.032	3.034	3.033	0.0012
19	3.048	3.053	3.056	3.052	0.0040
20	3.034	3.046	3.064	3.048	0.0151
21	3.004	3.012	3.004	3.007	0.0046
22	2.934	2.955	2.956	2.948	0.0124
23	3.094	3.093	3.091	3.093	0.0015
24	3.018	3.009	3.012	3.013	0.0046
25	3.016	3.010	3.015	3.014	0.0032
26	2.977	2.957	2.958	2.964	0.0113
27	3.023	3.030	3.022	3.025	0.0044
28	3.041	3.032	3.022	3.032	0.0095
29	3.036	3.029	3.022	3.029	0.0070

Data collected on November-2-2002

Model check data collected for the **BLUE car** by reading the times off the large display of the Microwizard, Fast Track timer appears below.

Configuration	Run 1	Run 2	Run 3	Run 4	Run 5	Ave.	St.Dev.
30	2.852	2.852	2.846	2.853	2.838	2.848	0.0063

Data collected on November-2-2002

Model check data collected for the **YELLOW car** by reading the times off the large display of the Microwizard, Fast Track timer appear below. As during the trial, the car sported very thin wheels.

Configuration	Run 1	Run 2	Run 3	Run 4	Run 5	Ave.	St.Dev.
30	2.868	2.858	2.879	2.868	2.871	2.869	0.0075

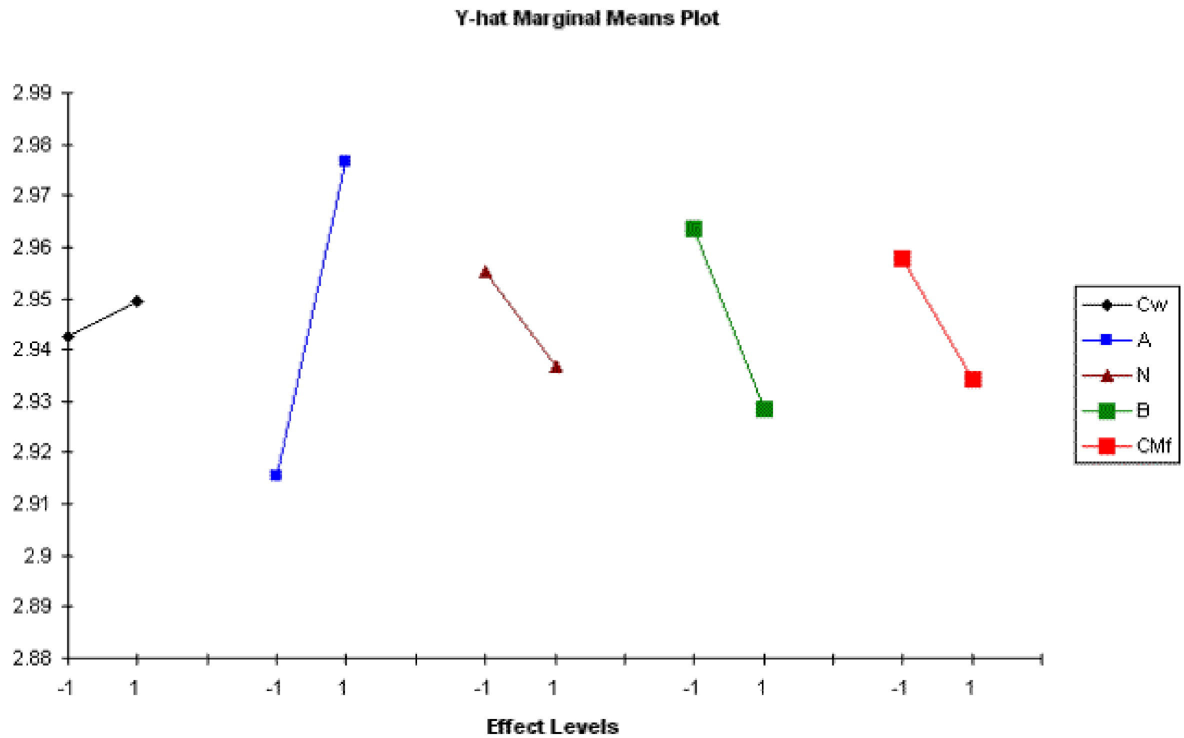
Analysis

From the data a statistical model was constructed for both the average time (Y-hat model) and race time variation (S-hat model) of both time trial cars. The actual times and variations for each configuration were subtracted from the predictions of these models to determine how far off they were. If the models are good, the actual measurements and variation are highly correlated with the model predictions. The yellow car data was twice as noisy as the blue, but both still provided a significant average time model. Only the blue car had a significant time variation model.

Randy's software created 4 plots: *Y-Hat* a plot of effect on average race time; *Y-Hat Pareto* an ordered view of half Y-Hat factor spreads; *S-Hat* the effect on race time variation; and the ordered *S-Hat Pareto* view. The spreads in the pareto charts are coefficients for models *coded* to the -1, +1 factor settings. Factor normalizations replaced these codes and the model equations were regrouped for use with actual factor values. The actual value models are presented below.

To study non-deterministic effects in the actual data, a deterministic prediction model derived from physical considerations produced a DOE style model for comparison. First, the two models must agree on stable configurations, then the remaining configurations can be analyzed for patterns and reasonable explanations. These educated guesses form the basis for further studies.

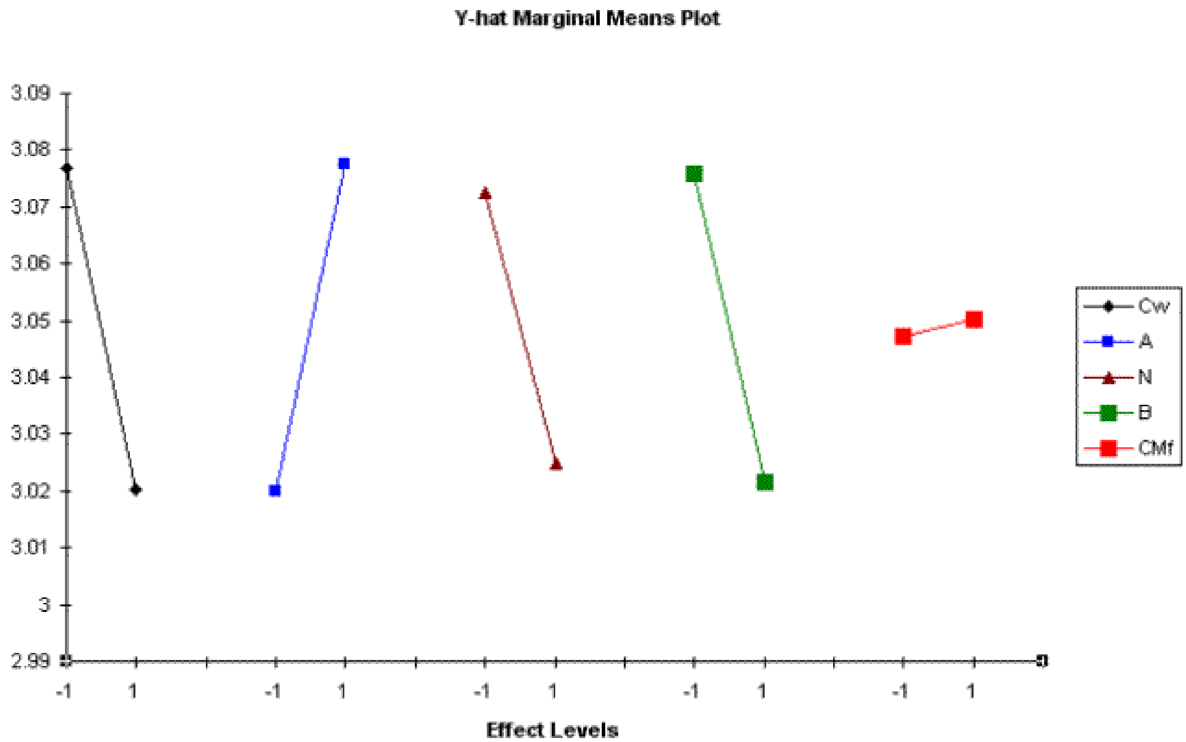
Y-Hat Marginal Means Plot for Blue Car:



The Y-hat Marginal Means Plot for the blue car graphically shows the effect of setting changes in the factor, low (-1) and high (1) on race time (y-axis). Let's look at the wheelbase "B". The low value (-1) on the x-axis corresponds to 3 7/8 inches, which has a race time of 2.964 seconds. The high value (1), 4 7/8 inches, (next point to the right on the x-axis) has a race time of 2.928 seconds. A line spans them. The larger wheelbase setting has a lower race time. This factor was found to have an effect on race time corresponding to about 5.5 inches at the finish line.

Based on this plot of time factors, less frontal area, more nose, more wheelbase and rearward center of mass led to faster runs for the yellow car.

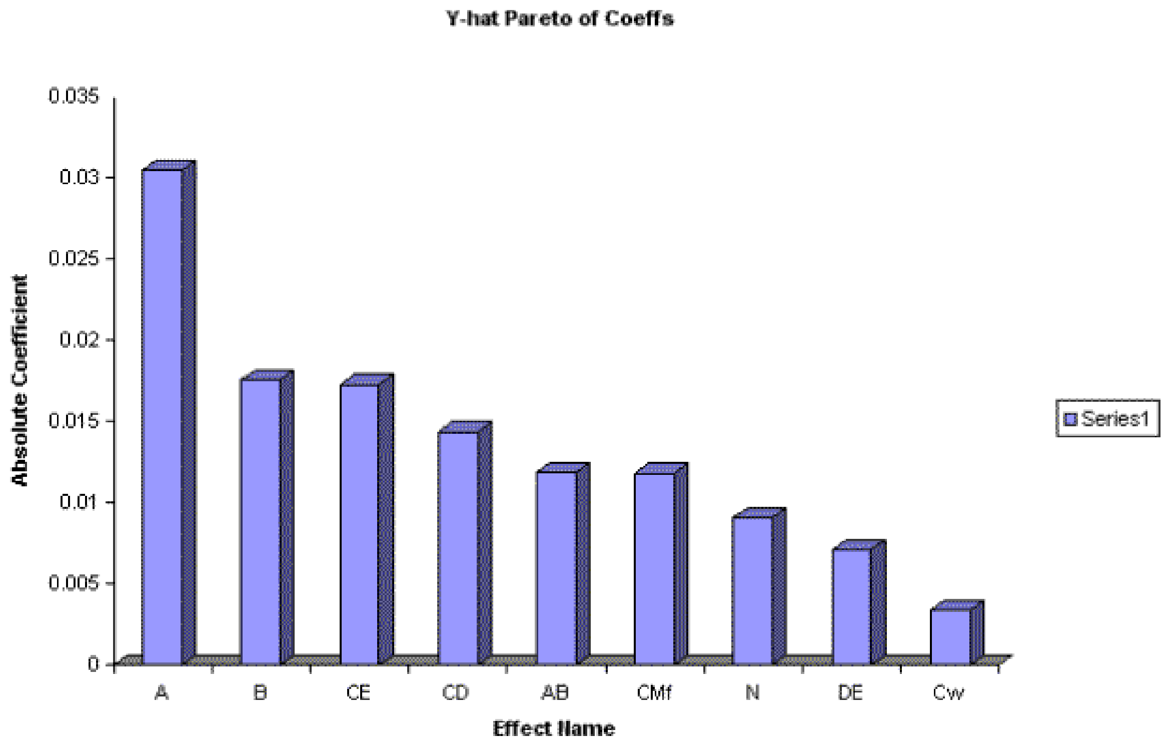
Y-Hat Marginal Means Plot for Yellow Car:



The Y-hat Marginal Means Plot for the yellow car scales differently. Yellow car frontal cross-section spans a time spread about the same as blue, but the other factors span about twice the time range including wheel / body clearance. This factor of two corresponds to the doubling of time variation in the yellow car data compared to blue. Wheel / body clearance for blue, like center of mass for yellow, is insignificant and reversed in direction of effect compared to the other car.

Based on this plot of time factors, more wheel-body separation, less frontal area, more nose and more wheelbase led to faster runs for the yellow car. But because of greater time variance, this formula for success in not as reliable as that found for the blue car.

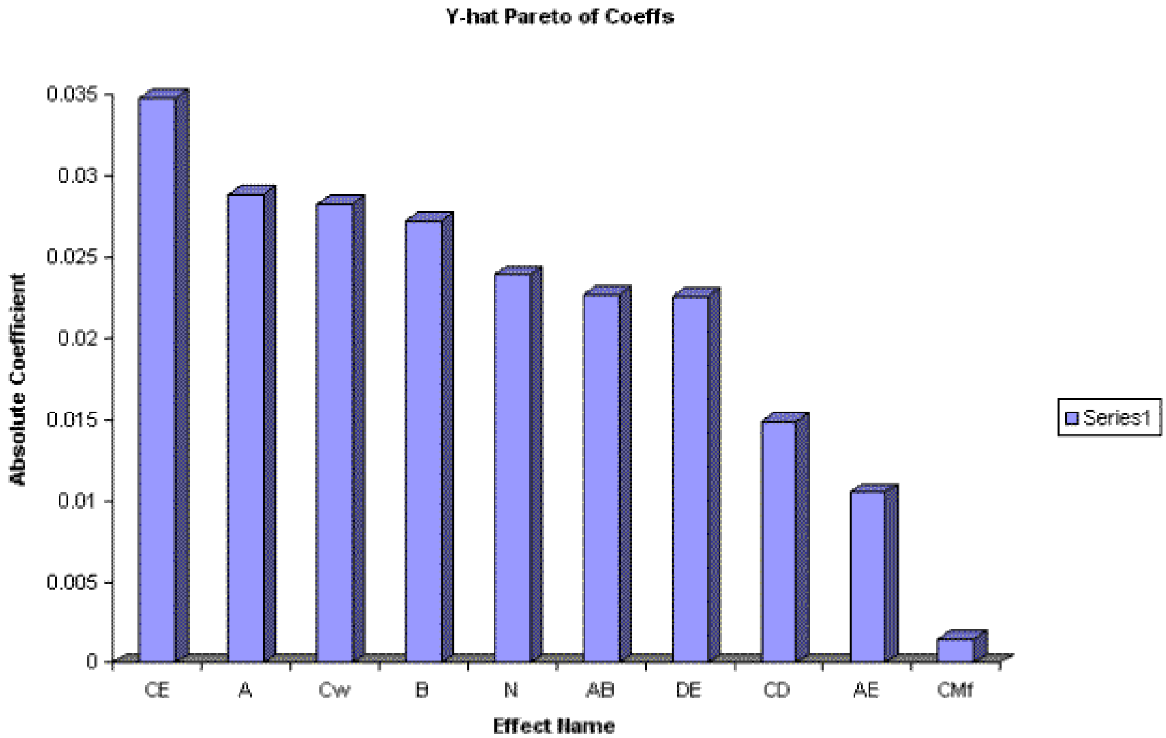
Y-Hat Pareto of Coefficients for Blue Car:



The Y-Hat Pareto of Coefficients chart plots half the y-axis spread from the Y-Hat Marginal Means Plot in order of largest spread to smallest. In this chart, it is easy to see which factors had the greatest effect on race time - but you can't tell whether the effect slowed the car down or sped it up. Notice the combined factors. "CE" is the joint effect of N and CMf. "CD" is N and B. "AB" is the effect of Cw and A. "DE" is B and CMf. These mixed factors indicate the combined effects on time of two factors beyond their effects acting alone.

One would think that frontal cross-section (A), center of mass (CMf) and wheel clearance (Cw) should not influence other factors to affect race time. Yet this plot shows they did. It makes sense that nose length, wheelbase and center of mass should pair-wise couple as they have been observed to affect stability together.

Y-Hat Pareto of Coefficients for Yellow Car:



The Y-Hat Pareto of Coefficients chart for the yellow car is in many ways similar to the that of the blue car. All but the frontal cross-section are about twice the blue values. Note "AE" is Cw and CMf. Many factors line-up in a different order, but order is not generally significant when the height is as close as these. Once again, these charts show CE and CMf effect one car but not the other.

Y-Hat Model for Blue Car:

1) average run time = -2.8889856 Cw - 0.004112 A - 0.174524286 N - 0.088083429 B + 0.080274286 CMf + 0.608256 Cw A + 0.05752 N B - 0.026285714 N CMf - 0.010788571 B CMf + 3.251547914

With a significant correlation value of 0.94, one expects the Blue average time model to estimate one of its seed 16 configurations, like number 10, very closely. Indeed the calculated time falls short of the actual average by only 6 thousandths of a second, -0.19% error. The "zero" configuration doesn't fair as well, over-estimating by 4 hundredths of a second; 1.21% error. However, the model predicted configuration 30 just 2 thousandths of a second under actual; -0.08% error. Not bad! This model fails to account for about 12% of the observed variance in the data after adjusting for small samples.

The model shows the five factors have both individual effects and combined effects. A, with a coefficient of -0.004112, actually decreases the race time when increased! However, for this blue car, A affects wheel play and slows the car down by an order of magnitude greater; 0.608256 Cw A. Yet, if the wheel / body gap could be set essentially to zero, this model indicates that the Cw A term would be negligible. Then increasing A would speed the car up! Even if the wheels had enough room to turn, few would believe this prediction. This model has more of these kinds of limitations.

None of the five factors have primary or mixed coefficients that are always one sign or the other. So no definitive statement like "increasing such and such a factor always decreases time" can be made. Regions of factor settings that decrease time must be found.

Some questions about the sensitivity of race time to some factors can be explored. For example, by taking the partial derivative of the average time with respect to nose length, setting the change in time to zero to find critical values and making a table using the factor values for B and CMf, the following can be stated. Race time is insensitive to N when CMf is about 2.2(B - 3 inches). So, if the CMf for a car with a typical B of 3.875 inches is 1.9 inches, measured from the tip of the nose, then the nose length won't affect race time. Conversely, race time is most affected by adjusting N when CMf is large and B is small. If N increases, race time decreases. Let's see what happens when B is changed.

The factors affecting race time sensitivity to B are CMf and N. Setting the partial differential equation to zero, obtains CMf = 5.33(N - 1.5). Solving this equation for CMf intersecting with CMf = 2.2(B - 3), leads to an equation for N, N = 0.4128 B + 0.25. For a given B, this equation finds an N such that small changes in N or B won't affect race time significantly. For a typical B of 3.875 inches, N would be 1.85 inches and CMf over the front axles at 1.86 inches. These are not optimal values, but small design changes from them won't change the performance of the car.

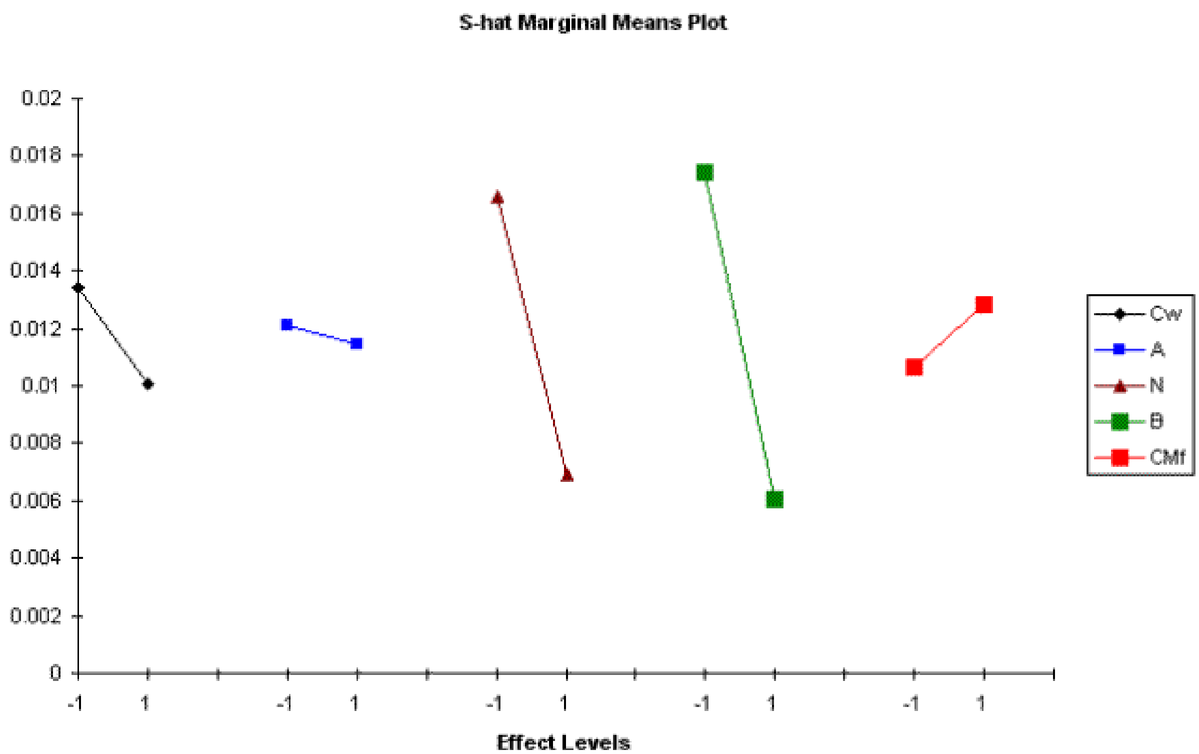
Y-Hat Model for Yellow Car:

2) average run time = -5.8741504 Cw - 0.031272 A - 0.116066667 N - 0.025225333 B + 0.260169905 CMf + 1.160704 Cw A - 0.512 Cw CMf + 0.05952 N B - 0.053013333 N CMf - 0.034346667 B CMf + 3.081572795

The Yellow time average model cozies up to within 1 to 2 hundredths of a second of its seed configurations, but over-predicts configuration 30 by a whopping 0.19 seconds; 6.69% error. This result jibes with the model's lower adjusted correlation, 0.90, though significant, and its portrayal of CMf behavior opposite to that of the Blue model. CMf in configuration 30 rests as far to the rear as possible without making the time trial car unstable. This model fails to account for about 18% of the observed variance in the data.

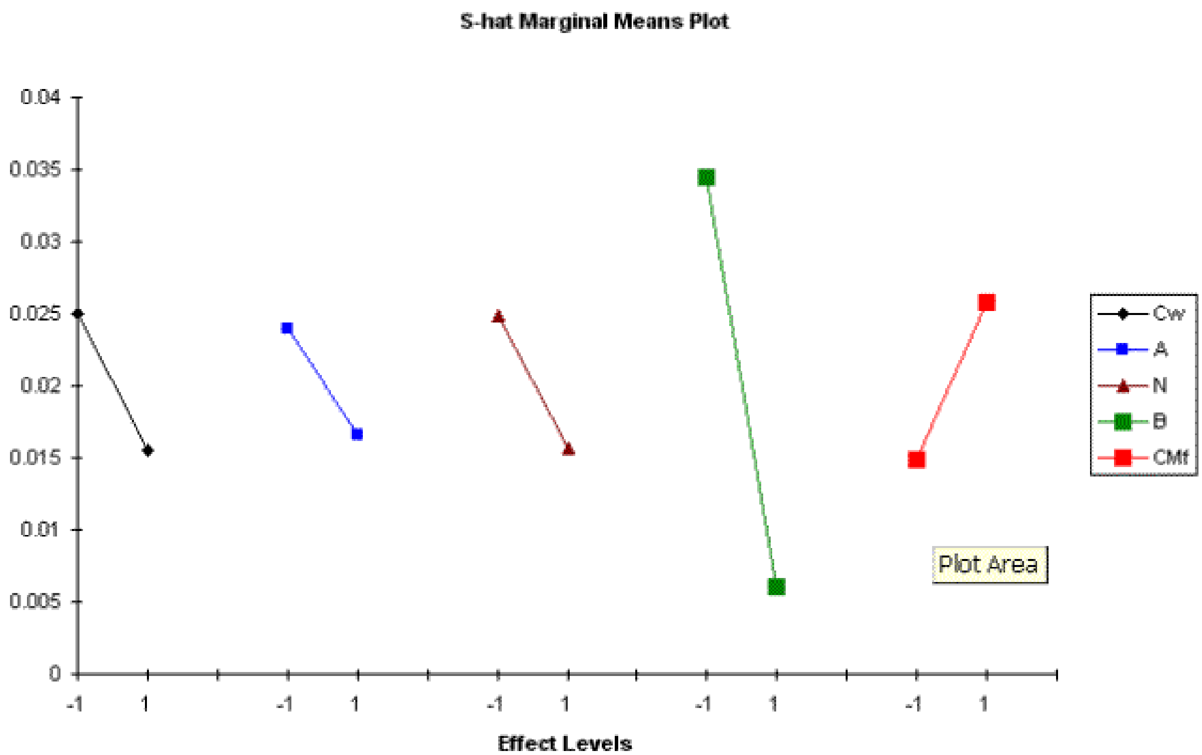
The signs of the yellow model coefficients agree with those of the blue model. However, 6 out of 10 coefficient magnitudes are two times or more than the corresponding blue ones since twice as much variation is incorporated. Note the added Cw CMf term.

S-Hat Marginal Means Plot for Blue Car:



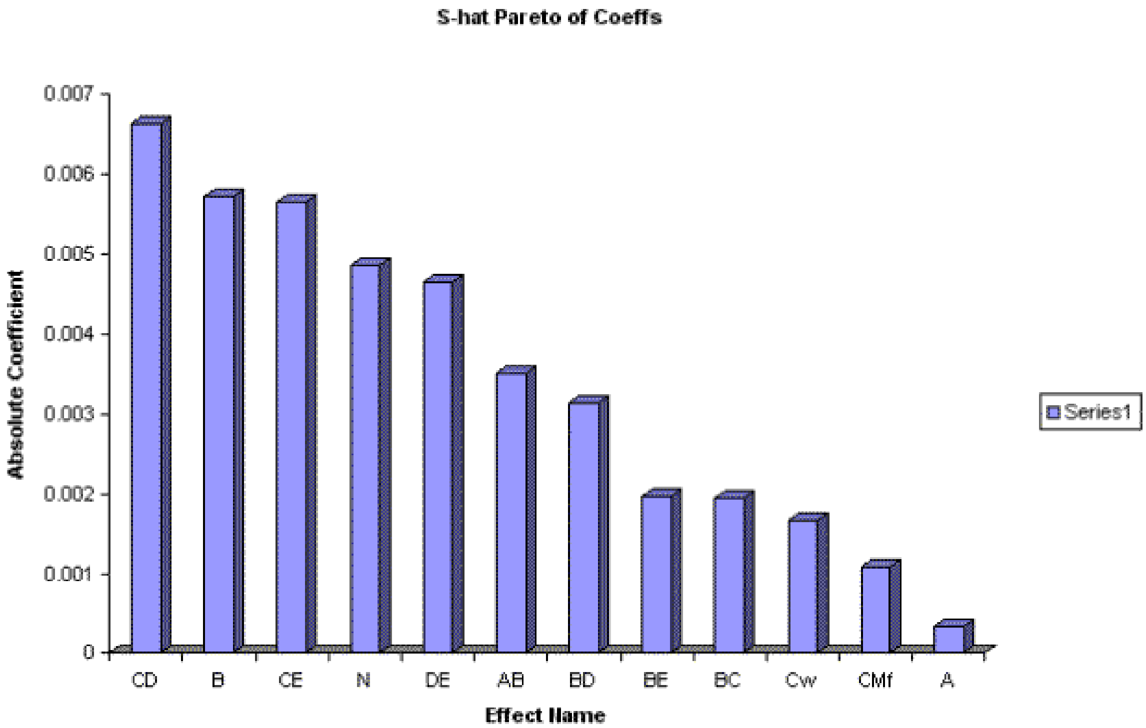
Y-axis values on the S-Hat Marginal Means Plot center around 0.01175 seconds which is more-or-less an average deviation for all blue car race time measurements. The deviation in the measurements resulting from each factor being low and then high is plotted as a line. The lower S-hat values indicate less variation in race time. These values give clues as to which factors help "stabilize" a pinewood car. Significant factors have a spread greater than the S-hat center value (0.01175). Champion cars must increase stability for top speed. This S-Hat Marginal Means Plot for the blue car shows that a long nose and wheelbase may reduce race time variation.

S-Hat Marginal Means Plot for Yellow Car:



Y-axis values on the plot center around 0.02028 seconds. This is almost exactly the same value as achieved in the DOE screening experiment. Again, longer wheelbase reduces race time variation, but not necessarily a longer nose.

S-Hat Pareto of Coefficients for Blue Car:



The S-Hat Pareto of Coefficients chart plots half the y-axis spread from the S-Hat Marginal Means Plot in order of largest spread to smallest. It is easy to see which factors had the greatest effect on race time variation - but you can't tell whether the effect was stabilizing or destabilizing. For the blue car, CD (N B), B, and CE (N CMf) show significant effects on race time variation. N and DE (B CMf) may also. These are all mixed and direct effects of N, B and CMf. When mixed terms show up, they indicate an effect over and above that of the factors acting alone. Just think about what happens when the weight is too far forward or rearward; the car wheelies. With the front or rear set of wheels hovering over the track, the car has no "rudder" and snakes. It may rub the top of the median and not even finish the race.

Note, measurement of N and CMf overlap since their measurement origin is the same; the front of the car. But B is measured from where N leaves off. When B is set to "short", and CMf is "aft", the weight can be close to the rear axle or more toward the center, depending on whether N was "short" or "long". A change in B changes the location of CMf over the wheelbase that depends on N, whereas the location of CMf with respect to N doesn't change. I suspect this produced the wider span in the CE (N CMf) line than for DE (B CMf).

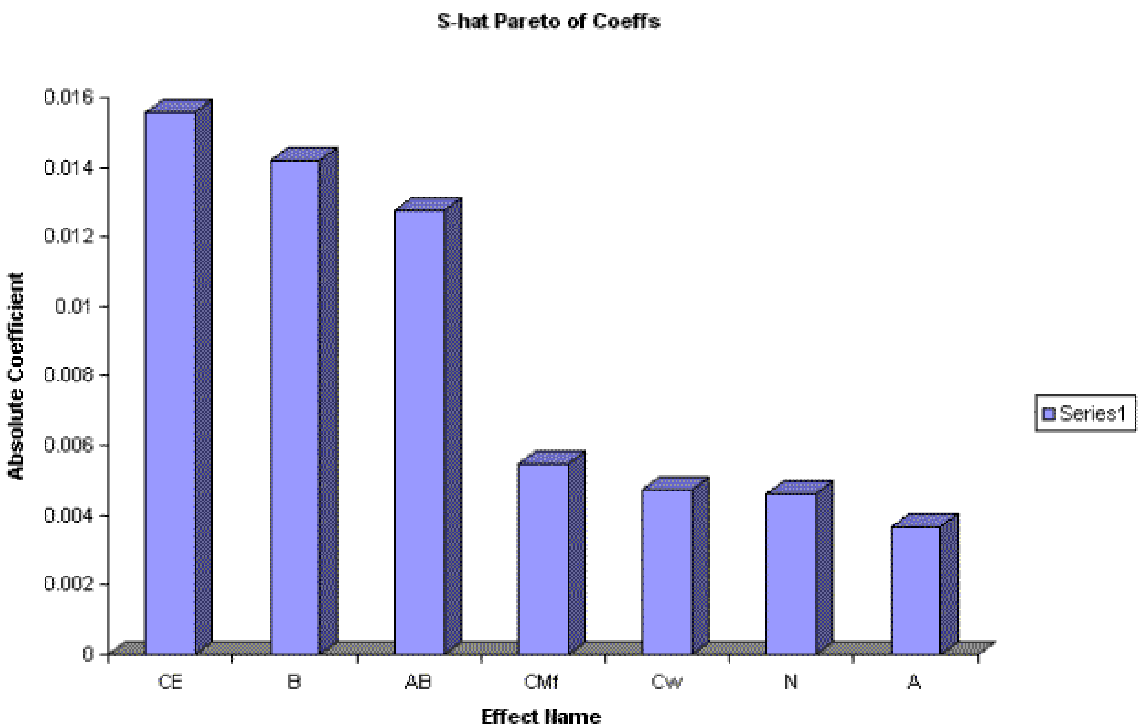
A related study [3] shows that road noise varies inversely with wheelbase and directly with speed. Road noise undoubtedly affects race time variation. Therefore, any factor that affects race time should affect the time variation. All five factors appear, though all are not significant. Note A is last in the chart. However, AB (Cw and A), BC (A and N), BD (A and B) and BE (A and CMf) also appear. They show that vibrations from pressure drag act through the lever arms offered by the nose and base around CMf, subject to 'play' between the wheels and the body. Wheelbase provides the greatest leverage and therefore has a long bar, BD. It is curious that the wheel play has a greater effect in AB. The way (+increase or -decrease) in which these mixed factors affect time variation can be seen in the terms of the S-Hat model below.

S-Hat Model for Blue Car:

3) time variation = -1.0181888 Cw - 0.03108912 A - 0.110044752 N - 0.053497486 B + 0.051642133 CMf + 0.178688 Cw A + 0.003088 A N + 0.004992 A B - 0.001194667 A CMf + 0.02644 N B - 0.008579048 N CMf - 0.007085714 B CMf + 0.260792677

The S-hat model produced from the blue car time data proved significant. With a correlation coefficient of 0.988, this model only leaves 2.4% of variation unaccounted for in the seed data. However, application to other configurations produce wide margins of error. This may be because the seed configurations were limited to the first 16 configurations.

S-Hat Pareto of Coefficients for Yellow Car:



Note the yellow car B and paired factors of N and CMf as well as Cw and A had significant stabilizing interactions.

Additional "sanity checks" were made using ANOVA (analysis of variance) analysis. From ANOVA analysis, we get the *standard error* of the data, the *Fischer ratio* (F), and another measure of significance. The standard error measures how accurately the mean (average) race times were determined by the experiment. It was very small, so the means are very accurate. The Fisher Ratio indicates if there are any factors in our model that are significant. It won't pinpoint which ones. When F is greater than 6, there is likely a significant factor. F for both the average time models and blue race time variation model was well above 6, confirming that something was indeed significant. The third measure is the probability that none of the factors were significant (Sig F). This one was very near zero, confirming again that likely there were no insignificant factors.

Physical Model Comparison:

One goal of this study seeks to compare the theoretical, closed model of a pinewood race[4] to real race data. An attempt to symbolically compare the DOE Modeling equations, 1 and 2 to the theoretical one proved too daunting. A computer program like Mathematica[5] would be needed to form a linearized approximation in a reasonable amount of time. Instead, the RaceIt[6] simulation program based on an unpublished derivation of the theoretical model proved useful.

All 30 of the trial car configurations were modeled in RaceIt XML input files along with the track. RaceIt simulates about 20 deterministic car parameters and a few more for the track and environment. Cw, the wheel clearance is not a deterministic factor and was included in the input files but not used. The other four time trial factors were set with the values indicated in this report. Other physical factors were easily measured from the time trial cars, except for axle and tread friction coefficients. These latter two were initially set to typical values and adjusted to reduce the average model and actual time differences. Reasonable values for the Blue car surfaced, but the Yellow car required higher values than expected. It may be that the very thin wheels used with the Yellow car created more friction through instabilities of their own.

Text output from the RaceIt program passed through a JavaScript converting it to XML. An XSL stylesheet compared the Modeling DOE data to the RaceIt data. Immediately it was obvious that some configurations lead to wide differences in the predicted and actual times failing to account for 37% of the variability in the data. The time trial logs recorded observation of various instabilities for most of them. Snaking and related motions result from "stochastic" or random events that detract from the energy of the car. These too fast RaceIt Blue predicted times give evidence that these configurations might be subject to more random events than others. Since the RaceIt prediction model is deterministic, the same 8 configurations were dropped from the Blue and Yellow data. These configurations, 1, 3, 8, 9, 12, 13, 15 and 28 give a first pass at quantifying the effects of random processes in pinewood racing below.

Results were very encouraging for the Blue car data. RaceIt predicted times for the time trial configurations with only 6.7% of the variability unaccounted for, a correlation of 0.966 and standard error of estimate 0.009. The Yellow car data could not account for 31% of the variability with a correlation coefficient of 0.829 and standard error of estimate of 0.032. Though not very accurate, this correlation was significant.

Evidence for instability:

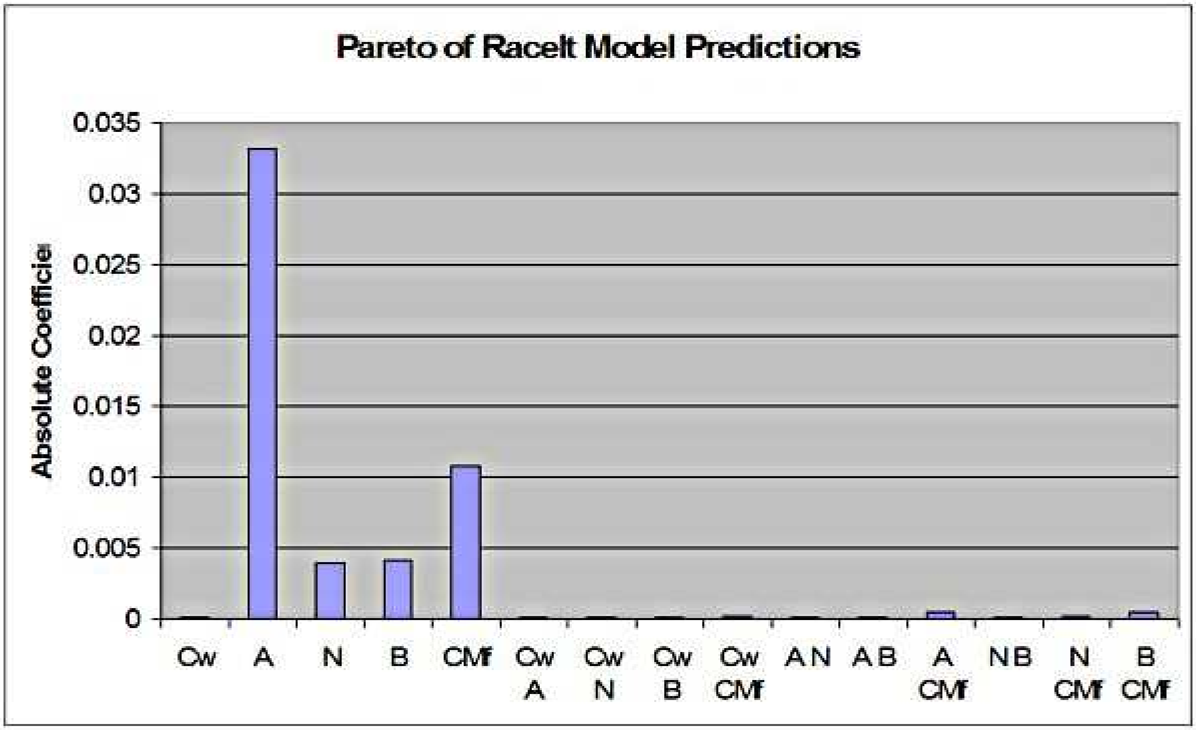
What can we learn from configurations, 1, 3, 8, 9, 12, 13, 15 and 28? Is there some systematic reason that the actual times were slower? 5 of 8 of their standard deviations were the highest in the data set except for configuration 5. *All configurations with a short nose and wheelbase had high run time variation*; configurations 1, 5, 9 and 13. High variation in run times indicates some form of random drift.

Two more configurations, 3 and 15, have a long nose, short wheelbase and forward CMf. Configurations 8 and 12 have a long nose and wheelbase with forward CMf. Front CMf also proved problematic in configuration 28. The remaining configurations with forward CMf, 2 and 14, which also had a short nose and long wheelbase, were predictable with low deviation in run times. The evidence suggests that *forward weighting exhibits slower actual times than predicted from deterministic factors unless the nose is short and wheelbase is long*.

Because of heat and stochastic energy losses, the RaceIt predicted times should all be equal to or faster than the actual times as above. However, small slower time differences up to 3 standard deviations from the actual time would not invalidate the model. Using the average standard deviation for the actual Blue car data, 0.0037 seconds, 3 "sigma" is above one hundredth of a second. Blue

RaceIt predictions that were *too slow* by more than a hundredth of a second occurred for the following configurations: 6(-0.013), 19(-0.013) and 21(-0.016). All race time variations were very small in this group. Note that RaceIt treated 17 through 21 identically producing the same time, but only 19 and 21 are problematic. Perhaps small sample error, three runs each, explains all three slow time predictions.

Y-Hat Pareto of Coefficients for RaceIt Predictions:



Treating the RaceIt race time predictions as 2-Level data using only the first 16 configurations, a Y-Hat Pareto of Coefficients above results. Note it is not sorted. Only A, N, B and CMf are significant. A and CMf hold very closely to the Blue car average time pareto values, but N and B contribute substantially less to the RaceIt predictions. The hypothesis that differences in N, B and the various cross terms portray contributions to various instabilities in the Blue car was tested as described below. This idea makes sense since RaceIt is deterministic. As desired, the DOE method produced a simple equation for the blue car's RaceIt predicted times. It is surprising that the complex, non-linear RaceIt model for the blue car can be very well approximated by the linear function [7] below.

Linearized RaceIt Model for Blue Car:

4) predicted time = 2.8423766 + 0.02655 A - 0.007875 N - 0.0041875 B - 0.0082381 CMf

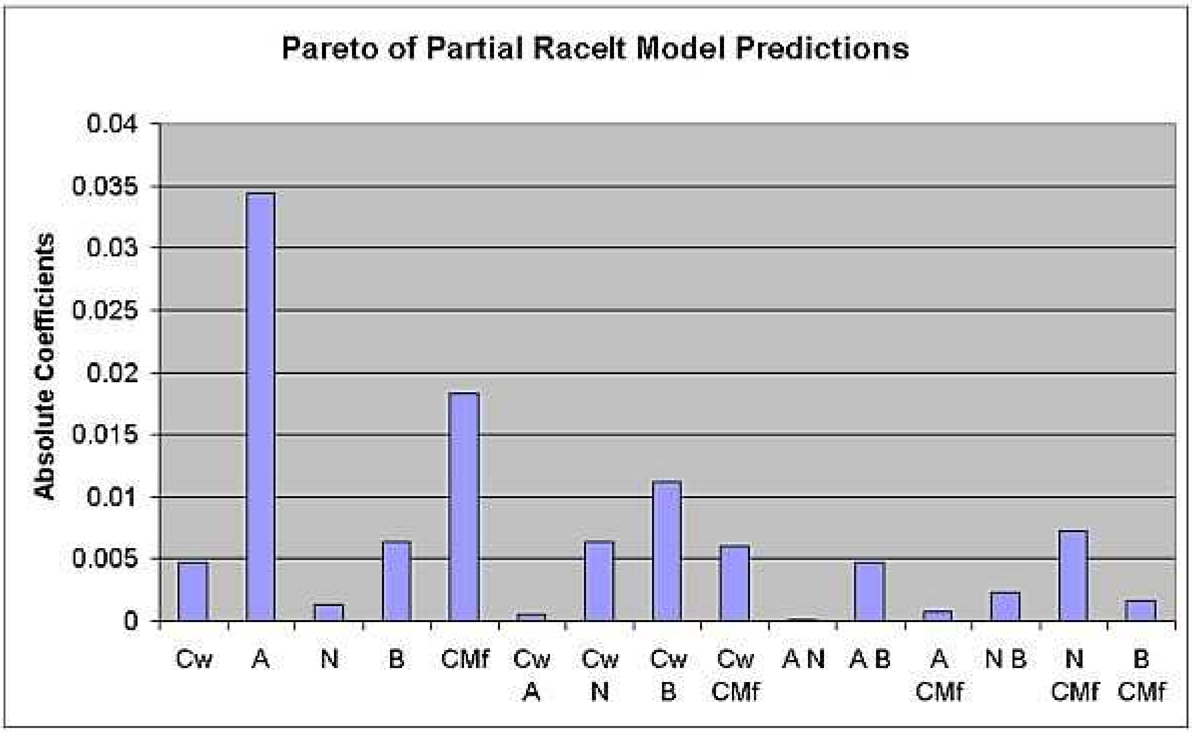
Results

Non-deterministic behavior:

Comparing deterministic theory to actual pinewood run data helped identify variation in run times due to non-deterministic behavior. This behavior has only a few causes through four interfaces, air flow and car, tread and track, lane median and inner wheel surface, and wheel / axle / body. Pressure from air flow, roughness of the track and lane median, poor alignment of interfaces, roughness in materials and off center or out-of-round wheels power the gyrations of non deterministic behavior.

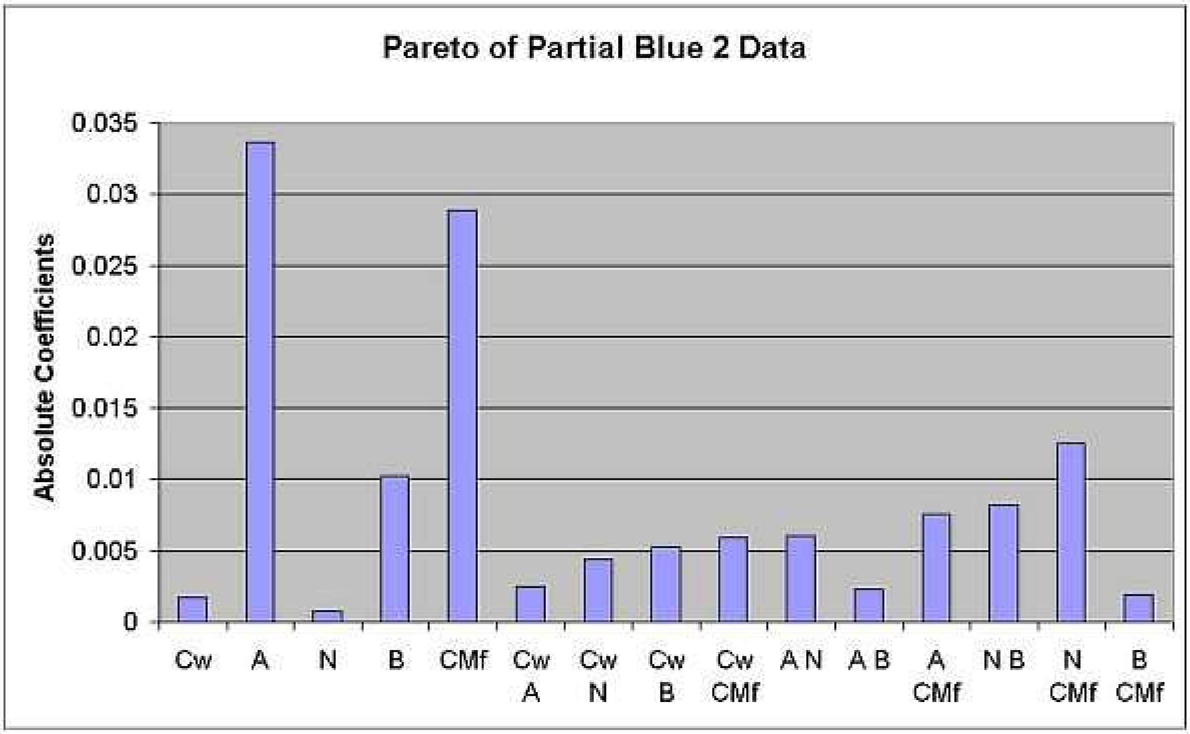
The wheel bore pits itself against the axle and a small body surface; the only car parts in relative motion. Through this interface, replicated four times, forces act on the body and the wheels competing for control of the car's path. The resulting chaos, collision, shaking and rubbing rob energy from the car and increase the time needed to arrive at its destination. We found it necessary to tame this interface just to get data good enough to produce any significant results.

Removing noisy configurations 1, 3, 12, 13 and 23 presents:



To test the hypothesis that variation due to instability caused the differences in the bar lengths of the blue car data and the RaceIt predictive pareto charts, we removed the noted unstable configurations that had the highest standard deviations from chart production. Witness the result for the RaceIt predictions above and that for the Blue car data below. The two charts now appear much more alike and the hypothesis seems plausible.

Pareto for the blue car data without noisy configurations:



However, limiting the data in this way biases the model equations produced and they become useless for actual prediction though more highly correlated to each other. These mathematical theatrics demonstrate that *the blue car closely follows deterministic physical behaviors as long as it is stable*. Unstable configurations become sensitive to irregularities in the track, the air and the wheel bore interfaces amplifying them through various levers (N and B) joints (Cw) and around fulcrums (CMf). All of these effects show up in the mixed terms of the blue average time model and more so in the yellow car model.

An energy budget sketch:

One way to assess which areas of a system need improvement is to look at where the energy goes. The RaceIt program allows us to obtain measurements of energy usage that would be very difficult to measure in an experiment. The energy budget for each virtual race includes major energy sinks and accounts for at least 99% of the total potential energy. Since the RaceIt predictions using all 30 configurations could not account for 37% of the variation in the blue car data, that percentage forms an upper bound on the variation due to instabilities. The table below normalizes the RaceIt energy budget for blue configuration 30 to 63% to show a complete, sorted breakdown. It is not likely that configuration 30 would lose so much energy to instabilities, so this budget may simulate a "bad" case scenario, like wheels coming out of alignment.

RaceIt energy budget for Blue 30 on a really bad day:

Type of Energy Used	Percent	
Linear Kinetic Energy	47.77	
Instabilities	37	
Aerodynamic drag	7.49	
Wheel inertia	1.65	
	Front	Rear
Tread friction	0.53	3.44
Axle friction	0.24	1.88

One wants to increase linear kinetic energy because that translates into speed. The other energy sinks are not desirable. Arguably, this energy budget may not be valid but it stresses the importance of stabilizing a pinewood car with respect to other major energy sinks that competitors may spend more time reducing.

The analysis of this experiment suggests that instabilities may be reduced in the following ways though some may not be statistically significant.

1. Decrease A
2. Increase B
3. Increasing both N and CMf (CE) together reduces the time. However, the next factor combination which is almost as important, needs N small. So increase CMf.
4. Decreasing both N and B (CD) together reduces the time. If one is increased, the other must be decreased. Short B is definitely bad. So shorten N to as close to 0 as possible.
5. Decreasing both Cw and A (AB) together reduces the time. If one is increased, the other must be decreased. Decreasing Cw too much can jam the wheel, so there must be an optimal point.
6. Increase both B and CMf (DE) together
7. Larger Cw makes stability problems worse

Further Investigation

Comparing DOE method results with good predictive models has proven a useful pattern for future investigations. Yet, the author believes the current experiment can be improved upon and clearer results obtained. Some factor ranges can be expanded and slightly redefined.

Coded factor ranges:

The DOE method produces *coded* model coefficients; the height of the pareto bars. The codes for each factor are the integers -1 and +1 representing low and high settings. The range or span of the settings directly affects the height of the pareto bars. Some speed factors like cross-section have a large range, 2.5 square inches, compared to others like wheel / body separation, 0.125 inches. Though legal wheelbases range up to 4.6 inches, to accommodate all configurations with the other 4 factors without wheelying, its span could only be 1 inch (2.375 inches for -2 and +2 codes). Likewise, the nose length spans only 1 inch and the center of mass 2 5/8 inches. To obtain statistically significant results, it is desirable that the coded factors span as wide a range as possible. Allowing some factors to exceed legal race limits may allow three-level modeling. The effect of factor range on the model coefficients is corrected when the codes are replaced by their actual value normalizations and regrouped. All time model equations in this report were transformed for actual factor values.

Changing factor definitions:

In the design of this experiment, the coupling between N, B and CMf proved rather complex. Using a different definition of these factors may isolate them to a greater extent. For example, N locates the front axle, Xf, and N + B locates the rear axle, Xr. Using these "locations" instead of N and B produces a direct substitution of Xf for N and Xf - Xr for B. If the code N=1 B=1 is replaced by Xf=1 Xr=1 and N=-1 B=-1 by Xf=-1 Xr=-1, then for N=-1 B=1 and N=1 B=-1, Xf -1 and 1 can be used, but there is no corresponding polar code for Xr though the two configurations specify the same Xr location. That position would not be included in a two-level model, but code 0 would work in a three-level.

This "middle" value was included indirectly in the current two-level configurations (1 to 16). In this sense, more variation was designed into the experiment than desired. Eliminating the middle position allows a greater range for Xr-Xf above what was possible using B. In essence, the effect of B should show up as the coefficient of Xf Xr combined.

CMf could also be changed to CMr, measuring the center of mass from the rear axle instead of from the front of the car. Here we have the opposite case from that above. The coupling between the rear axle location and CMr would introduce two new implicit positions for CM since their ranges are different. We did something right, do not use CMr in any of your time trials!

Notes

[1] Lastufka, Michael, DOE Screening Time Trial Pinewood Grand Prix, 16 March 2002, Lastufka Labs, Michael_Lastufka@sil.org available on request.

[2] Purchase options included a computer link, but we did not order it. The timer was purchased for Lastufka Labs by Dan Kolsar and Jeff Heath at a special price. Thanks Microwizard, it's a great timer!

[3] Tamboli, J. A., S. G. Joshi, Optimum Design of a Passive Suspension System of a Vehicle Subjected to Actual Random Road Excitations, *Journal of Sound and Vibration* (1999) 219(2), 193-205. Article No. jsvi.1998.1882, available online at http://www.idealibrary.com

[4] Lastufka, Michael, Tracking Down Solutions, 1997, Lastufka Labs, http://www.worldforchrist.org/races/cars/why/summry.htm.

[5] Wolfram Research, Mathematica, http://www.Wolfram.org/.

[6] Lastufka, Michael, Race It! Users' Guide, Version 1.0 for DOS, 1997, Lastufka Labs, http://www.worldforchrist.org/races/cars/why/raceit.htm. Free download from the User's Guide.

[7] The equation obtained using the RaceIt pareto coefficients was corrected by subtracting 0.0799 seconds. This offset was the average time difference between the raw model times and the RaceIt times due to dropping the smaller factors and combinations. The correlation coefficient is 0.994 with a standard error of 0.004.